

Table 23.1 Future Value of \$1

Yield rate = 10.0%

Amount of original deposit = \$1.00

	1 + Interest Rate	×	Previous Balance	=	New Balance
End of Year 1	1.1000	×	\$1.0000	=	\$1.1000
End of Year 2	1.1000	×	\$1.1000	=	\$1.2100
End of Year 3	1.1000	×	\$1.2100	=	\$1.3310
End of Year 4	1.1000	×	\$1.3310	=	\$1.4641
End of Year 5	1.1000	×	\$1.4641	=	\$1.6105
End of Year 6	1.1000	×	\$1.6105	=	\$1.7716
End of Year 7	1.1000	×	\$1.7716	=	\$1.9487
End of Year 8	1.1000	×	\$1.9487	=	\$2.1436
End of Year 9	1.1000	×	\$2.1436	=	\$2.3579
End of Year 10	1.1000	×	\$2.3579	=	\$2.5937

Table 23.2 Present Value of \$1

Yield rate = 10%

Year					Present Worth
1	\$1.00	/	110.0%	=	\$0.9091
2	\$0.91	/	110.0%	=	\$0.8264
3	\$0.83	/	110.0%	=	\$0.7513
4	\$0.75	/	110.0%	=	\$0.6830
5	\$0.68	/	110.0%	=	\$0.6209
6	\$0.62	/	110.0%	=	\$0.5645
7	\$0.56	/	110.0%	=	\$0.5132
8	\$0.51	/	110.0%	=	\$0.4665
9	\$0.47	/	110.0%	=	\$0.4241
10	\$0.42	/	110.0%	=	\$0.3855

Table 23.3 Discounted Cash Flows—Level Annuity

Discount rate = 10%

Year	Cash Flow	Discount Factor*	Present Value
1	\$100,000	0.9091	\$90,909
2	\$100,000	0.8264	\$82,645
3	\$100,000	0.7513	\$75,131
4	\$100,000	0.6830	\$68,301
5	\$100,000	0.6209	\$62,092
6	\$100,000	0.5645	\$56,447
7	\$100,000	0.5132	\$51,316
8	\$100,000	0.4665	\$46,651
9	\$100,000	0.4241	\$42,410
10	\$1,100,000	0.3855	\$424,098
Total			\$1,000,000

* Figures in column rounded to four significant digits for display.

Table 23.4 Investment Analysis of Drugstore

Discount rate for the periodic cash flows = 8%

Year	Cash Flow	×	Discount Factor	=	Present Value
1	\$190,674	×	0.92593	=	\$176,550
2	\$190,674	×	0.85734	=	\$163,472
3	\$190,674	×	0.79383	=	\$151,363
4	\$190,674	×	0.73503	=	\$140,151
5	\$190,674	×	0.68058	=	\$129,770
6	\$190,674	×	0.63017	=	\$120,157
7	\$190,674	×	0.58349	=	\$111,256
8	\$190,674	×	0.54027	=	\$103,015
9	\$190,674	×	0.50025	=	\$95,384
10	\$190,674	×	0.46319	=	\$88,319
Present value of the periodic cash flows only					\$1,279,438
Discount rate for the reversion = 14%					
10	\$2,060,000	×	0.26974	=	\$555,672
Total					\$1,835,110

Figure 23.1 Income Stream Patterns

Pattern	Characteristics	Valuation Techniques Used
Variable annuity—nonsystematic change	An income stream that is not regular or predictable.	It is very difficult to analyze with any technique but discounted cash flow (DCF) analysis.
Level annuity	An income stream that is the same each period like a mortgage payment.	This income can be valued by DCF analysis or direct capitalization depending on the reversion value.
Ordinary annuity	Payments are in arrears like a mortgage.	Most real estate investments are valued as if in arrears.
Annuity payable in advance	Payments are made in advance of the period. Annuity in advance is like a lease contract that says payments are made in advance.	The present value of an ordinary annuity with payments in arrears can be converted to the present value in advance by multiplying the arrears number by $1 + i$. For example, if the present value of \$15,000 per year for 10 years discounted at 8% in arrears is \$100,651, the present value in advance would be $\$100,651 \times 1.08$ or \$108,703.32. This can be confirmed on any handheld calculator by changing it from “End” to “Begin” mode.

Increasing or decreasing annuities

Step up or step-down annuities	Usually leases that have steps up or down for a variety of reasons. These can be a result of expense increases or amortization of tenant improvements.	The only practical technique to value these income streams is the DCF model.
Straight-line (constant-amount) change per period annuity	A cash flow that is increasing or decreasing on a straight-line basis (i.e., not compounded). The amount of change is the same dollar amount each period. This is like a lease that calls for the rent to go up \$500 per year every year over the life of the lease.	These income streams can be valued by direct capitalization but are more easily valued by discounted cash flow analysis.
Exponential-curve (constant-ratio) change per period annuity	A cash flow that is going up or down on a compound basis. This is like a lease agreement that states the rent goes up 3% each year compounded annually.	This income stream can be valued by direct capitalization or using the DCF model.

Table 23.5 Straight-Line Change

Discount rate = 9.00%

Year	Cash Flow		Discount Factor	=	Present Value
1	\$1.00	×	0.9174	=	\$0.91743
2	\$1.05	×	0.8417	=	\$0.88376
3	\$1.10	×	0.7722	=	\$0.84940
4	\$1.15	×	0.7084	=	\$0.81469
5	\$1.20	×	0.6499	=	\$0.77992
6	\$1.25	×	0.5963	=	\$0.74533
7	\$1.30	×	0.5470	=	\$0.71114
8	\$1.35	×	0.5019	=	\$0.67752
9	\$1.40	×	0.4604	=	\$0.64460
10	\$1.45	×	0.4224	=	\$0.61250
11	\$1.50	×	0.3875	=	\$0.58130
12	\$1.55	×	0.3555	=	\$0.55108
13	\$1.60	×	0.3262	=	\$0.52189
14	\$1.65	×	0.2992	=	\$0.49376
15	\$1.70	×	0.2745	=	\$0.46671
16	\$1.75	×	0.2519	=	\$0.44077
17	\$1.80	×	0.2311	=	\$0.41593
18	\$1.85	×	0.2120	=	\$0.39219
19	\$1.90	×	0.1945	=	\$0.36953
20	\$1.95	×	0.1784	=	\$0.34794
21	\$2.00	×	0.1637	=	\$0.32740
22	\$2.05	×	0.1502	=	\$0.30787
23	\$2.10	×	0.1378	=	\$0.28934
24	\$2.15	×	0.1264	=	\$0.27177
25	\$2.20	×	0.1160	=	\$0.25513
26	\$2.25	×	0.1064	=	\$0.23938
27	\$2.30	×	0.0976	=	\$0.22450
Total				=	\$14.13278

Table 23.6 Exponential-Curve Change

Discount rate = 9.00%

Year	Cash Flow		Discount Factor		Present Value
1	\$1.00	×	0.9174	=	\$0.91743
2	\$1.04	×	0.8417	=	\$0.87535
3	\$1.08	×	0.7722	=	\$0.83519
4	\$1.12	×	0.7084	=	\$0.79688
5	\$1.17	×	0.6499	=	\$0.76033
6	\$1.22	×	0.5963	=	\$0.72545
7	\$1.27	×	0.5470	=	\$0.69217
8	\$1.32	×	0.5019	=	\$0.66042
9	\$1.37	×	0.4604	=	\$0.63013
10	\$1.42	×	0.4224	=	\$0.60122
11	\$1.48	×	0.3875	=	\$0.57364
12	\$1.54	×	0.3555	=	\$0.54733
13	\$1.60	×	0.3262	=	\$0.52222
14	\$1.67	×	0.2992	=	\$0.49827
15	\$1.73	×	0.2745	=	\$0.47541
16	\$1.80	×	0.2519	=	\$0.45360
17	\$1.87	×	0.2311	=	\$0.43280
18	\$1.95	×	0.2120	=	\$0.41294
19	\$2.03	×	0.1945	=	\$0.39400
20	\$2.11	×	0.1784	=	\$0.37593
21	\$2.19	×	0.1637	=	\$0.35868
22	\$2.28	×	0.1502	=	\$0.34223
23	\$2.37	×	0.1378	=	\$0.32653
24	\$2.46	×	0.1264	=	\$0.31155
25	\$2.56	×	0.1160	=	\$0.29726
26	\$2.67	×	0.1064	=	\$0.28363
27	\$2.77	×	0.0976	=	\$0.27061
Total				=	<u>\$14.37121</u>

Table 23.7 Level Income in Perpetuity

Analysis of property with level income and no change in value at 8%

Direct capitalization = $\$25,000/0.08 = \$312,500$

Year	Income		Discount Factor	=	Present Value
1	\$25,000	×	0.925926	=	\$23,148
2	\$25,000	×	0.857339	=	\$21,433
3	\$25,000	×	0.793832	=	\$19,846
4	\$25,000	×	0.735030	=	\$18,376
5	\$25,000	×	0.680583	=	\$17,015
6	\$25,000	×	0.630170	=	\$15,754
7	\$25,000	×	0.583490	=	\$14,587
8	\$25,000	×	0.540269	=	\$13,507
9	\$25,000	×	0.500249	=	\$12,506
10	\$25,000	×	0.463193	=	\$11,580
11	\$25,000	×	0.428883	=	\$10,722
12	\$25,000	×	0.397114	=	\$9,928
13	\$25,000	×	0.367698	=	\$9,192
13*	\$312,500	×	0.367698	=	<u>\$114,906</u>
Net present value					<u>\$312,500</u>

* Reversion

Table 23.8 Analysis of Property with Level Income and Change in Value

$\Delta = 10\%$

$Y = 0.10$

$N = 5 \text{ years}$

$R = Y - (\Delta - 1/S_n)$

$R = 0.10 - (0.10 \times 0.16380)$

$R = 0.083620$

Direct capitalization: $\$25,000/0.083620 = \$298,972$

Year	Cash Flow		Discount Factor		Present Value
1	\$25,000	×	0.909090909	=	\$22,727
2	\$25,000	×	0.826446281	=	\$20,661
3	\$25,000	×	0.751314801	=	\$18,783
4	\$25,000	×	0.683013455	=	\$17,075
5	\$25,000	×	0.620921323	=	\$15,523
5*	\$328,869	×	0.620921323	=	\$204,202
Net present value					\$298,972

* The reversion was calculated as 125% of the present value or $\$298,972 \times 1.10 = \$328,869$

Table 23.9 Constant Change in Income and Value

Increase in income and value = 35.00%

Income = \$35,000

Holding period = 10 years

$Y = 0.08$

$R = Y - (\Delta \times 1/n)$

$R = 0.08 - (0.35 \times 0.10)$

$R = 0.08 - 0.035$

$R = 0.045$

Direct capitalization: $\$35,000/0.045 = \$777,778$

To calculate the amount of cash flow change each period, the following formula applies:

$V \times (\Delta 1/n) \times Y =$ the increase in cash flows

$\$777,778 \times (0.35 \times 0.10) \times 0.08 =$ the increase in cash flows

$\$777,778 \times 0.035 \times 0.08 = \$2,177.78$

Year	Initial Income		Income Change		Adjusted Income		Discount Factor	=	Present Value
1	\$35,000				\$35,000	x	0.9259259	=	\$32,407
2	\$35,000	+	\$2,178	=	\$37,178	x	0.8573388	=	\$31,874
3	\$35,000	+	\$4,356	=	\$39,356	x	0.7938322	=	\$31,242
4	\$35,000	+	\$6,533	=	\$41,533	x	0.7350299	=	\$30,528
5	\$35,000	+	\$8,711	=	\$43,711	x	0.6805832	=	\$29,749
6	\$35,000	+	\$10,889	=	\$45,889	x	0.6301696	=	\$28,918
7	\$35,000	+	\$13,067	=	\$48,067	x	0.5834904	=	\$28,046
8	\$35,000	+	\$15,244	=	\$50,244	x	0.5402689	=	\$27,146
9	\$35,000	+	\$17,422	=	\$52,422	x	0.5002490	=	\$26,224
10	\$35,000	+	\$19,600	=	\$54,600	x	0.4631935	=	\$25,290
10	\$1,050,000						0.4631935	=	\$486,353
Net present value									<u>\$777,778</u>

Table 23.10 Constant-Ratio Change in Income and Value

$$R = Y - CR$$

$$R = 0.08 - 0.03$$

$$R = 0.05$$

Direct capitalization: $\$35,000/0.05 = \$700,000$

Year	Income	Reversion	Total Cash Flow	Discount Factor	Net Present Value
1	\$35,000.00		\$35,000	0.92592593	\$32,407.41
2	\$36,050.00		\$36,050	0.85733882	\$30,907.06
3	\$37,131.50		\$37,132	0.79383224	\$29,476.18
4	\$38,245.45		\$38,245	0.73502985	\$28,111.54
5	\$39,392.81		\$39,393	0.68058320	\$26,810.08
6	\$40,574.59		\$40,575	0.63016963	\$25,568.88
7	\$41,791.83		\$41,792	0.58349040	\$24,385.13
8	\$43,045.59		\$43,046	0.54026888	\$23,256.19
9	\$44,336.95		\$44,337	0.50024897	\$22,179.51
10	\$45,667.06		\$45,667	0.46319349	\$21,152.69
11	\$47,037.07		\$47,037	0.42888286	\$20,173.39
12	\$48,448.19		\$48,448	0.39711376	\$19,239.44
13	\$49,901.63		\$49,902	0.36769792	\$18,348.73
14	\$51,398.68		\$51,399	0.34046104	\$17,499.25
15	\$52,940.64		\$52,941	0.31524170	\$16,689.10
16	\$54,528.86	\$1,123,295	\$1,177,823	0.29189047	\$343,795.41
Total =					\$700,000.00

Table 23.11 Constant-Ratio Change in Income and Value

Year	Income	Reversion	Total Cash Flow	Discount Factor	Net Present Value
1	\$35,000.00		\$35,000.00	0.92592593	\$32,407.41
2	\$36,050.00		\$36,050.00	0.85733882	\$30,907.06
3	\$37,131.50		\$37,131.50	0.79383224	\$29,476.18
4	\$38,245.45		\$38,245.45	0.73502985	\$28,111.55
5	\$39,392.81		\$39,393.81	0.68058320	\$26,810.08
6	\$40,574.59		\$40,574.59	0.63016963	\$25,568.88
7	\$41,791.83		\$41,791.83	0.58349040	\$24,385.13
8	\$43,045.59		\$43,044.59	0.54026888	\$23,256.19
9	\$44,336.95		\$44,336.95	0.50024897	\$22,179.51
10	\$45,667.06		\$45,667.06	0.46319349	\$21,152.69
10		\$940,741.47*		0.46319349	<u>\$435,745.32</u>
					<u>\$700,000.00</u>

* $\$700,000 \times 1.03^{10} = \$940,741.47$